Sketching Sine and Cosine Curves

$$y = C + A \sin B (x - D)$$

$$y = C + A \cos B (x - D)$$

A = Amplitude (vertical height from centerline)
B = $\frac{2\pi}{period} = \frac{360°}{period}$ (the period is the length)
C = Centerline
D = Displacement (horizontal shift)

EXAMPLE 2

Sketch the graph of

$$y = -2 + 4 \sin \frac{\pi}{5}(x - 3).$$

Solution:
The period is

$$p = \frac{2\pi}{\frac{\pi}{5}} = 10.$$

Notice that when B is a multiple of π, the π's will cancel, and the period will *not* be a multiple of π. In this case, the x-axis can be marked in more familiar units.

The sinusoidal axis is at $y = -2$. The upper and lower bounds are 4 units above and below this sinusoidal axis. A cycle starts at $x = 3$, the phase displacement. Since this is the *sine* function, the graph starts at a *mid-point*, going *up*. The end of this cycle is at $x = (3 + 10) = 13$. Again, there are critical points each $\frac{1}{4}$-cycle. The graph is shown in Figure 13-6g.

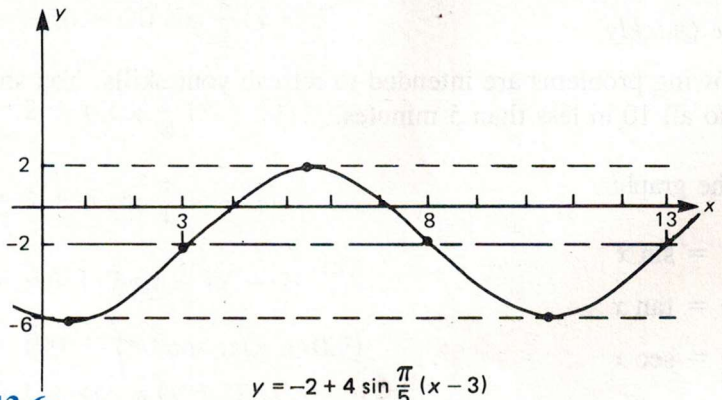


Figure 13-6g

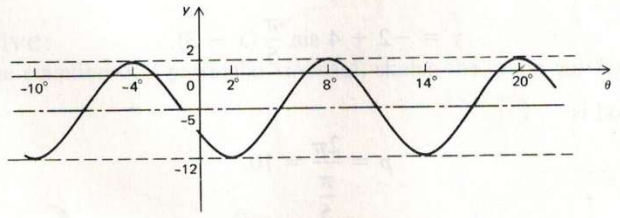
EXAMPLE 3

Sketch the graph of the trigonometric function $y = -5 + 7 \cos 30(\theta + 4°)$.

Solution:
The only way the trigonometric function graphs differ from the circular functions is that the horizontal axis is labeled in degrees. The period is

$$p = \frac{360°}{30} = 12°.$$

Since the phase displacement is $-4°$, the graph starts with a high point at $\theta = -4°$, and has its next high point at $\theta = (-4 + 12)° = 8°$. The sinusoidal axis is at $y = -5$, and the amplitude is 7. The graph is shown in Figure 13-6h.



$y = -5 + 7 \cos 30(\theta + 4°)$

Figure 13-6h

The exercise that follows is designed to give you practice in sketching the graphs of sinusoids.

For Problems 1 through 14, sketch the graph of the indicated trigonometric or circular function. You may use different scales on the horizontal and vertical axes, if necessary, to make the graphs have reasonable proportions.

1. $y = 5 + 2 \cos 3(\theta - 20°)$
2. $y = 3 + 4 \cos 5(\theta - 10°)$
3. $y = -3 + 4 \sin 10(\theta + 5°)$
4. $y = -1 + 3 \sin 12(\theta + 6°)$
5. $y = 3 + 2 \cos \frac{1}{5}(x - \pi)$
6. $y = 7 + 3 \cos \frac{1}{4}(x - 3\pi)$
7. $y = -4 + 5 \sin \frac{2}{3}\left(x + \frac{\pi}{2}\right)$
8. $y = -5 + 4 \sin \frac{1}{3}\left(x + \frac{\pi}{2}\right)$
9. $y = -10 + 20 \cos \frac{\pi}{3}(x - 1)$
10. $y = 2 + 6 \cos \frac{\pi}{4}(x - 3)$
11. $y = 3 + 5 \sin \frac{\pi}{4}(x - 3)$
12. $y = -6 + 7 \sin \frac{\pi}{4}(x - 2)$